The only real root is

$$x = \frac{v^2}{ag} = 2r \sec^2 \beta.$$

In case the ball at the time of impact has passed the highest point of its path ($v^2 \sin 2\beta < 2ag$), the problem is clearly impossible; this may be made to appear by putting $(-C)$ in place of $(C)$ in the foregoing discussion.

There are two cases when $v^2 \sin 2\beta > 2ag$:

I. If

$$\frac{7}{2}\mu(1+e) > \tan \alpha_1, \quad v^2 = ag \csc 2\beta \left[\frac{7 + 10e + 7e^2}{e(7e+5)}\right].$$

II. If

$$\frac{7}{2}\mu(1+e) < \tan \alpha_1, \quad v^2 = ag \sec^2 \beta \frac{1+e}{2e(\tan \beta - \mu)}.$$

To these may be added:

III. If $\mu = 0$,

$$v^2 = ag \frac{1+e}{e} \operatorname{cosec} 2\beta.$$

By means of the relation

$$\tan \alpha_1 = \tan \beta - \frac{ag}{v^2} \sec^2 \beta = \tan \beta \left(1 - \frac{2ag}{v^2 \sin 2\beta}\right),$$

combined with the values of $v^2$ in the two cases, we find that Case I (rolling impact) or Case II (rolling and sliding at impact) occurs, according as $\mu \cot \beta$ is greater than or less than $[2(1-e)/7 + 10e + 7e^2]$.

If the ball is not homogeneous, the criterion for Case I and Case II is

$$\mu > \text{or} < \frac{k^2}{a^2 + k^2} \cdot \frac{\tan \alpha_1}{1+e},$$

and $K$ becomes $a^2/e(a^2 + k^2)$, where $k$ is the radius of gyration for a diameter. The discussion is otherwise unchanged, so that in this more general case

$$v^2 = ag \operatorname{cosec} 2\beta \cdot \frac{2a^2e + (a^2 + k^2)(1 + e^2)}{a^2e + (a^2 + k^2)e^2}$$

or

$$v^2 = ag \sec^2 \beta \cdot \frac{1+e}{2e(\tan \beta - \mu)},$$

according as $\mu \cot \beta$ is greater or less than

$$\frac{k^2(1-e)}{2a^2e + (a^2 + k^2)(1 + e^2)}.$$

Also solved by J. A. Caparo, A. M. Harding, and Joseph B. Reynolds.

Note.—No solution of 300 has been received. H. S. Uhler should have received credit for solving 297 and 298. Editors.

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# QUESTIONS AND DISCUSSIONS.

Edited by U. G. Mitchell, University of Kansas.

## NEW QUESTIONS.

**30.** A certain Normal University wishes to offer thirty-five hours of college mathematics for the benefit of high-school teachers. What should these courses be in order that, primarily, they may be of the greatest value to high-school teachers of mathematics and, secondarily, that they may furnish stimulus for a more extended pursuit of the subject?

Note.—In transmitting this question the proposer writes, "The mathematical courses of our colleges seem to be designed chiefly for two classes of students, those expecting to pursue the

subject either in the pure or the applied field. It is conceivable that these courses are not the best for high-school teachers of mathematics, especially for those who do not pursue their studies in course beyond the college. It is quite likely that there should be more survey courses even at the expense of intensive work over a narrow field. It is quite certain that the emphasis should be changed, giving more geometrical, historical and pedagogical courses. Hence I offer the above complement to Question No. 27."

Replies to Question No. 27 were published in the December and January issues of the MONTHLY.

**31.** In the light of questions 27 and 30, the Editors wish to propose that a symposium be called for on the question: What are the actual courses now offered in colleges and universities in this country for the preparation of teachers of mathematics (1) for secondary schools, (2) for colleges? The discussion may well lead to the consideration also of what courses *should be offered* for the preparation of teachers of mathematics (1) for secondary schools, and (2) for colleges.

## DISCUSSIONS.

### I. RELATING TO A SIMPLE PROOF BY INDUCTION OF AN INTERESTING NUMBER RELATION.

By CHARLES R. DINES, Dartmouth College.

THEOREM: *For any set* $(a_i \mid i = 1, 2, \cdots, n)$ *of* $n$ *distinct numbers, real or imaginary* $(n > 1)$, *we have*

$$\sum_{i=1}^{n} \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{1}{a_i - a_j} = 0. \tag{I}$$

*Proof:* (A) Relation (I) holds for the smallest admissible value of $n$, viz., $n = 2$.

(B) Assume that relation (I) holds when $n = r$; that is, for any set of $r$ distinct integers,

$$\sum_{i=1}^{r} \prod_{\substack{j=1 \\ j \neq i}}^{r} \frac{1}{a_i - a_j} = 0. \tag{1}$$

Let

$$\sum_{i=1}^{r+1} \prod_{\substack{j=1 \\ j \neq i}}^{r+1} \frac{1}{a_i - a_j} = N. \tag{2}$$

We wish to prove that $N$ is zero. Multiply both sides of (1) by $1/(a_{r+1} - a_1)$ and add to (2). Then

$$N = \sum_{i=1}^{r+1} \prod_{\substack{j=1 \\ j \neq i}}^{r+1} \frac{1}{a_i - a_j} + \frac{1}{a_{r+1} - a_1} \sum_{i=1}^{r} \prod_{\substack{j=1 \\ j \neq i}}^{r} \frac{1}{a_i - a_j}. \tag{3}$$

But

$$\prod_{j=2}^{r+1} \frac{1}{a_1 - a_j} + \frac{1}{a_{r+1} - a_1} \prod_{j=2}^{r} \frac{1}{a_1 - a_j} = 0$$

and when $1 < i < r + 1$

$$\prod_{\substack{j=1 \\ j \neq i}}^{r+1} \frac{1}{a_i - a_j} + \frac{1}{a_{r+1} - a_1} \prod_{\substack{j=1 \\ j \neq i}}^{r} \frac{1}{a_i - a_j} = \frac{a_{r+1} - a_1 + a_i - a_{r+1}}{(a_{r+1} - a_1)(a_i - a_{r+1})} \prod_{\substack{j=1 \\ j \neq i}}^{r} \frac{1}{a_i - a_j},$$